

Allometric Exponent and Randomness

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Abstract. An allometric height-mass exponent γ gives an approximative power-law relation $\langle M \rangle \propto H^\gamma$ between the average mass $\langle M \rangle$ and the height H , for a sample of individuals. The individuals in the present study are humans but could be any biological organism. The sampling can be for a specific age of the individuals or for an age-interval. The body-mass index (BMI) is often used for practical purposes when characterizing humans and it is based on the allometric exponent $\gamma = 2$. It is here shown that the actual value of γ is to large extent determined by the degree of correlation between mass and height within the sample studied: no correlation between mass and height means $\gamma = 0$, whereas if there was a precise relation between mass and height such that all individuals had the same shape and density then $\gamma = 3$. The connection is demonstrated by showing that the value of γ can be obtained directly from three numbers characterizing the spreads of the relevant random Gaussian statistical distributions: the spread of the height and mass distributions together with the spread of the mass distribution for the average height. Possible implications for allometric relations in general are discussed.

1. Introduction

Allometric relations in biology describe how a quantity Y scales with the body mass M , i.e., $Y = AM^{\frac{1}{\gamma}}$, where γ is an allometric exponent. Allometric relations have a long history with pioneering work by D'Arcy Thomson *On Growth and Form* [1] and J.S. Huxley *Problems of Relative Growth* [2]. Among others, the allometric relation for the metabolic rate B has drawn much interest: Kleiber's law [3] states that $B \sim M^p$, with $p = 1/\gamma \approx 3/4$, and has been tested in e.g. Refs. [4],[5], and [6]. For a review of allometric relations see Ref. [7] *Scaling in Biology*. In the present study we focus on the allometric relation between height and mass for humans. This mass (M) - height (H) relation has an even longer history going back to the pioneering work by A. Quetelet *A Treatise on Man and the Developments of his Faculties* from 1842 [8], where the allometric relation has been introduced to define a normal man so that M/H^γ becomes a Gaussian variable. The precise definition of the allometric exponent used in the present study is $\langle M \rangle_H \propto H^\gamma$ where $\langle M \rangle_H$ is the average mass of the individuals of height H in the sample. Note that the allometric exponents $\gamma = 1, 2$ and 3 correspond to that mass is proportional to height, body surface and body volume, respectively. For practical purposes $\gamma = 2$ is often a good approximation for humans, as shown in Ref. [9]. This approximation is the basis for the body mass index (BMI) A given by $\langle M \rangle_H = AH^2$, provided mass is in kilogram and height in meter. More recently it has been suggested that a larger allometric index $2 < \gamma \leq 3$ should be more appropriate [10, 11, 12]. In particular Burton in Ref. [11] suggests that $\gamma = 2$ is an underestimate caused by randomness. This is in accordance with the conclusions reached in the present investigation.

The object with the present investigation is to understand the relation between the exponent γ and the randomness for a given sample of individuals. The issue is best illustrated by a specific example. Figure 1(a) and (b) show the height and mass distributions, $P(H)$ and $P(M)$, respectively, for 25000 children 18 years old from Hong Kong [13]. Figure 1(c) in addition shows the distribution $P(M|H = \langle H \rangle)$ for the children of average height $\langle H \rangle$. All these three statistical distributions are to very good approximation Gaussians. This means that the variables in all three cases are randomly distributed around their respective average values. The random spread are in all three cases characterized by the normalized standard deviations, which we denote $\tilde{\sigma}_H$, $\tilde{\sigma}_M$, and $\tilde{\sigma}$ for random spread of height, mass and mass-for-average-height, respectively. The relation derived in the present paper states that γ to good approximation should be given by $\gamma = \frac{\sqrt{\tilde{\sigma}_M^2 - \tilde{\sigma}^2}}{\tilde{\sigma}_H}$. From the random spreads in Fig. 1 one then finds $\gamma = 1.63$. Figure 2(a) shows that this is a very accurate prediction. This means that the allometric exponent is entirely determined by the randomness of the three distributions. Why is this so and what does it imply? These are questions which come to mind.

In Sec. 2 the relation between γ and the random spreads is derived. Comparisons with data are made in Sec. 3, whereas we in Sec. 4 sum up and discuss the results.

2. Allometric exponent expressed in normalized standard deviations

The point made in the present paper is that the exponent γ can be estimated from the sole knowledge of the first and second moments of the mass and height distributions. In order to derive such a relation we assume that the mass and height distributions are approximately Gaussians. This is, as is illustrated by the datasets in Fig. 1, often a fair approximation around the maxima of the distributions. It means that the probability distribution for the mass and height distributions are approximately given by

$$P_M(M) = \frac{1}{\sqrt{2\pi\sigma_M^2}} \exp\left(-\frac{(M - \langle M \rangle)^2}{2\sigma_M^2}\right), \quad (1)$$

$$P_H(H) = \frac{1}{\sqrt{2\pi\sigma_H^2}} \exp\left(-\frac{(H - \langle H \rangle)^2}{2\sigma_H^2}\right), \quad (2)$$

respectively. Note that these two distributions are characterized by the four explicit numbers $\langle M \rangle$, $\langle H \rangle$, σ_M and σ_H . The degree of correlation between the mass and height is then given by the mass distribution for a given height, which we likewise assume to be approximately Gaussian and is given by the conditional probability

$$P(M|H) = \frac{1}{\sqrt{2\pi\sigma(H)^2}} \exp\left(-\frac{(M - \langle M \rangle_H)^2}{2\sigma(H)^2}\right), \quad (3)$$

where $\langle M \rangle_H$ and $\sigma(H)$ are the average and the standard deviation of mass obtained for all individuals of height H . Note that the standard deviation σ_X for a stochastic variable X is related to the first and second moments by $\sigma_X^2 = \langle X^2 \rangle - \langle X \rangle^2$. A particular feature in the present context is that the distribution of mass for a given height can approximately be characterized by a constant standard deviation σ , since in practice it turns out that $\sigma(H)$ is only weakly dependent on H in the close vicinity of $\langle H \rangle$ [see Fig. 1(d)]. Thus Eq. (3) can approximately be reduced to

$$P(M|H) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(M - \langle M \rangle_H)^2}{2\sigma^2}\right). \quad (4)$$

Another particular feature of the mass relation is that the average mass for a given height, $\langle M \rangle_H$, monotonously increases with height. We can use this one-to-one correspondence by changing the variable in Eq. (2) from H to $\langle M \rangle_H$. The height distribution in terms of $\langle M \rangle_H$ is then just $P_H(H(\langle M \rangle_H))$. This means that there exists a precise relation between the three distributions [Eqs. (1), (2), and (4)] given by

$$P_M(M) = \int d\langle M \rangle_H P(M|\langle M \rangle_H) P_H(H(\langle M \rangle_H)) \frac{dH(\langle M \rangle_H)}{d\langle M \rangle_H}. \quad (5)$$

Another crucial feature of the data is that $\langle M \rangle_H$ to some approximation is described by the power-law relation

$$\langle M \rangle_H = AH^\gamma. \quad (6)$$

This is the allometric relation in focus and discussed in the present paper. Here A and γ are two constants. The constant A can be expressed as $A = \langle M \rangle_H H^{-\gamma} = \langle M \rangle_{\langle H \rangle} \langle H \rangle^{-\gamma}$.

However, for Gaussian distributions $\langle H \rangle$ corresponds to the peak position of the height distribution and, since the individuals in this peak also to good approximation have the average mass, it follows that $\langle M \rangle_{\langle H \rangle} \approx \langle M \rangle$. In the following, we will consequently use the simplified estimate $A = \langle M \rangle \langle H \rangle^{-\gamma}$. The argument for P_H in Eq. (2) is

$$H(\langle M \rangle_H) = \left(\frac{\langle M \rangle_H}{A} \right)^{\frac{1}{\gamma}} = \langle H \rangle \left(\frac{\langle M \rangle_H}{\langle M \rangle} \right)^{\frac{1}{\gamma}}. \quad (7)$$

Close to the peak of this distribution at $\langle M_{\langle H \rangle} \rangle$ we can use the linear approximation

$$H(\langle M \rangle_H) = \langle H \rangle \left(\frac{\langle M \rangle_H}{\langle M \rangle} \right)^{\frac{1}{\gamma}} \approx \langle H \rangle \left(1 + \frac{1}{\gamma \langle M \rangle} (\langle M \rangle_H - \langle M \rangle) \right). \quad (8)$$

Inserting this approximation into the relation

$$P_H(\langle (M) \rangle) = P_H(H(\langle M \rangle_H)) \frac{dH(\langle M \rangle_H)}{d\langle M \rangle_H}$$

leads to the Gaussian distribution

$$P_H(\langle (M) \rangle) = \frac{1}{\sqrt{2\pi\sigma_H^2\gamma^2\langle M \rangle^2\langle H \rangle^{-2}}} \exp \left(-\frac{(\langle M \rangle_H - \langle M_{\langle H \rangle} \rangle)^2}{2\sigma_H^2\gamma^2\langle M \rangle^2\langle H \rangle^{-2}} \right). \quad (9)$$

Using Eq. (9) together with Eq. (4), means that the right-hand side of Eq. (5) becomes a convolution of two Gaussian. Since the convolution of two Gaussians with standard deviations σ_1 and σ_2 becomes a Gaussian with standard deviation $\sigma_3 = \sqrt{\sigma_1^2 + \sigma_2^2}$, it follows that $\sigma_M^2 = \sigma^2 + \sigma_H^2\gamma^2\langle H \rangle^2/\langle M \rangle^2$ or equivalently

$$\gamma = \frac{\langle H \rangle}{\langle M \rangle} \frac{\sqrt{\sigma_M^2 - \sigma^2}}{\sigma_H} = \frac{\sqrt{\tilde{\sigma}_M^2 - \tilde{\sigma}^2}}{\tilde{\sigma}_H}, \quad (10)$$

where we have introduced the normalized standard deviations $\tilde{\sigma}_M = \sigma_M/\langle M \rangle$, $\tilde{\sigma} = \sigma/\langle M \rangle$ and $\tilde{\sigma}_H = \sigma_H/\langle H \rangle$. Equation (10) is the central relation in the present investigation and shows that γ can be approximately obtained from the three dimensionless numbers $\tilde{\sigma}_M$, $\tilde{\sigma}$ and $\tilde{\sigma}_H$, which measures the random spread of the data in units of, respectively, the average mass and height of the individuals.

Also note that γ given by Eq. (10) is what you get when an allometric relation is used as an *ansatz*. It does not *a priori* say anything about whether or not an allometric relation is a good approximation of the data.

3. Comparison with data

In the light of the above theoretical underpinning we return to the data for 25000 children 18 years [13]. One notes in Fig. 1 that both the height and the mass distributions to good approximations are Gaussians for this dataset. The average mass for a child is $\langle M \rangle \approx 57.7$ kg and height $\langle H \rangle \approx 172.7$ cm. The standard deviations are $\sigma_M \approx 5.3$ kg and $\sigma_H \approx 4.8$ cm. Figure 1(c) shows that also the distribution of mass for given heights are Gaussians and Fig. 1(d) shows that the standard deviation $\sigma(H)$ in Eq. (3) is constant in a broad range of H around $\langle H \rangle$, so that Eq. (4) gives a very good description.

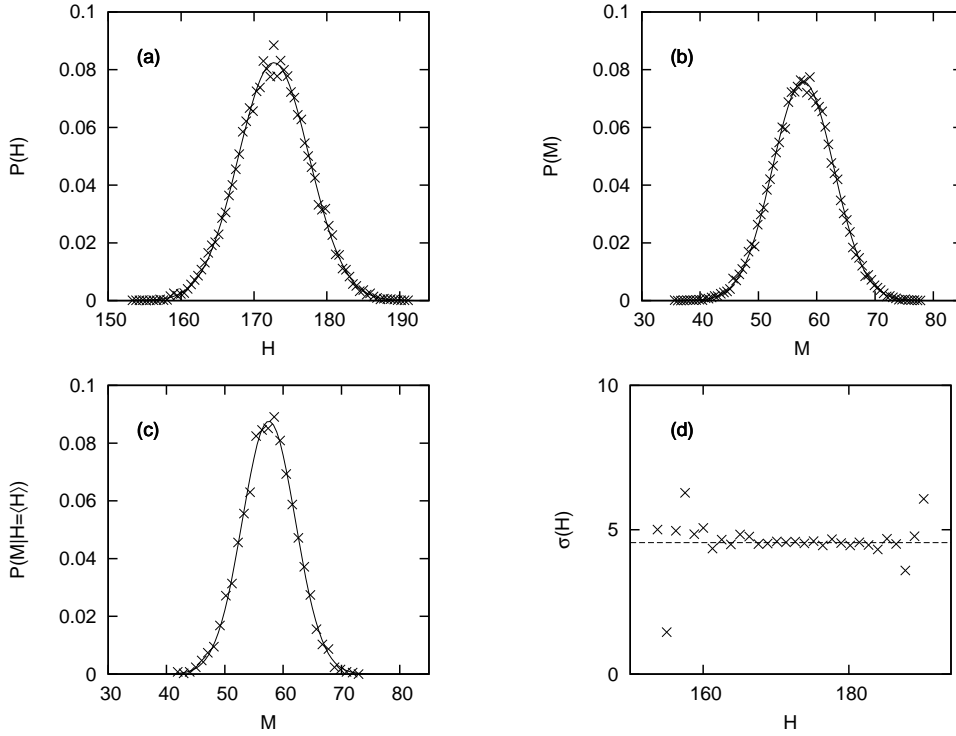


Figure 1. (a) The height distribution $P(H)$ and (b) the mass distribution $P(M)$ for 25000 Hong Kong children 18 years old [13]. (c) The conditional probability distribution $P(M|H = \langle H \rangle)$ obtained for 3670 children whose heights are in the interval $[171.82, 173.62]$ around $\langle H \rangle = 172.72$. In (a)-(c), the crosses are the data and the full-drawn curves are the corresponding Gaussian approximations. The numbers of bins are 81 for (a) and (b), and 31 for (c). H in (a) and M in (b) and (c) are in units of cm and kg. (d) The standard deviation $\sigma(H)$ of the distribution $P(M|H)$ in Eq. (3) as a function of height H . The horizontal line shows that the standard deviation $\sigma(H)$ is independent of H in a range around the average height $\langle H \rangle \approx 172.72$ cm.

As shown in Sec. 2, under these conditions the relation given by Eq. (10) applies. This relation states that if there is a power-law relation between average mass and height, $\langle M \rangle_H = AH^\gamma$, then the best prediction for the given information is

$$\langle M \rangle_H = \langle M \rangle \left(\frac{H}{\langle H \rangle} \right)^{\frac{\sqrt{\tilde{\sigma}_M^2 - \tilde{\sigma}^2}}{\tilde{\sigma}_H}} \quad (11)$$

Table 1 gives the average height and mass (in cm and kg, respectively) together with the three normalized standard deviations for the statistical distributions: $\tilde{\sigma}_H$, $\tilde{\sigma}_M$ and $\tilde{\sigma}$. The resulting power-law exponent predicted by $\gamma_{th} = \frac{\sqrt{\tilde{\sigma}_M^2 - \tilde{\sigma}^2}}{\tilde{\sigma}_H}$, as well as γ_{ex} obtained by direct fitting to the data [see Fig. 2(a)], is also listed. The agreement between γ_{th} and γ_{ex} is very precise and confirms that there really exists a relation between the spreads and the power-law exponent. The question is what it implies.

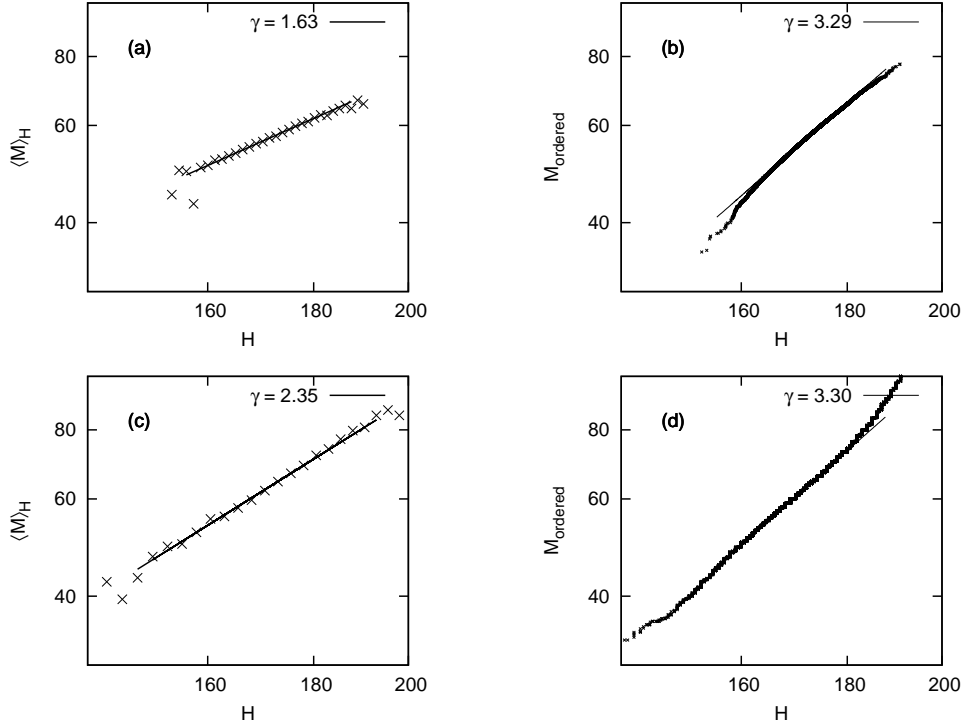


Figure 2. Allometric relations for the Hong Kong data in (a) and (b) and for the Swedish data in (c) and (d): (a) Log-log plot of the average mass $\langle M \rangle_H$ as a function of height H . Symbols correspond to the average for a length interval of 1.26 cm. The data fall on a straight line in accordance with the allometric relation $\langle M \rangle_H \propto H^\gamma$. The value of γ determined by the least-square fit to the data is $\gamma_{ex} = 1.63$. The straight line is the prediction in Eq. (11) in terms of the random spreads given by Eq. (10). The prediction is very accurate for this dataset. (b) Log-log plot of the ordered data $M_{\text{ordered}}(H)$ as a function of H . The data is well represented by $M_{\text{ordered}} \propto H^\gamma$ with $\gamma_{\text{ordered}} = 3.29$. (c) and (d) is the same as (a) and (b) for the Swedish data. The straight lines are least square fits to the data giving, respectively $\gamma_{ex} = 2.35$ and $\gamma_{\text{ordered}} = 3.30$. These predictions are again in good agreement with the predictions given in Table 1. Note that the allometric exponent between the Hong Kong data set in (a) and the Swedish data set in (c) are significantly different, whereas they are almost identical for within the ordered data-representation given by (b) and (d). These features are explained in the present paper.

In order to get an idea of what this means we note that if $\sigma = 0$ then there exists a one-to-one function between M and H and according to Eq. (10), we get

$$\gamma = \frac{\tilde{\sigma}_M}{\tilde{\sigma}_H}. \quad (12)$$

Changing $\tilde{\sigma}$ in the Hong Kong children data to $\tilde{\sigma} = 0$ changes the prediction for γ_{th} from 1.63 to 3.26 (compare Table 1 and Fig. 3). We can test this prediction against the children data by re-ordering so that the children are assigned masses which strictly follow the heights of the children. For this re-ordered data, σ is indeed zero and as shown in Fig. 2(b) the slope for this re-ordered data is indeed again close to the prediction. This

| | $\langle H \rangle$ | $\langle M \rangle$ | $\tilde{\sigma}_H$ | $\tilde{\sigma}_M$ | $\tilde{\sigma}$ | γ_{th} | γ_{ex} | $\gamma_{\sigma=0}$ | $\gamma_{ordered}$ |
|-----------|---------------------|---------------------|--------------------|--------------------|------------------|---------------|---------------|---------------------|--------------------|
| Hong Kong | 172.72 | 57.68 | 0.0280 | 0.0912 | 0.079 | 1.63 | 1.63 | 3.26 | 3.29 |
| Sweden | 170.48 | 60.52 | 0.0573 | 0.1862 | 0.130 | 2.33 | 2.35 | 3.25 | 3.30 |

Table 1. Summary of 25000 data for children 18 years old from Hong Kong [13] (first row) and for 11300 data for Swedish children in the year interval 13.5-19 years old [14] (second row). The average height $\langle H \rangle$ and the average mass $\langle M \rangle$ are in units of cm and kg. The normalized dimensionless standard deviations for the height $\tilde{\sigma}_H$, the mass $\tilde{\sigma}_M$, and for the mass distribution at average height $\tilde{\sigma}$ are listed. The theoretical prediction γ_{th} from Eq. (10) and γ_{ex} , obtained from the least-square fit to the data presented in Figs. 2(a) and (c), are in good agreement. $\gamma_{\sigma=0}$ from Eq. (10) with $\sigma = 0$ and $\gamma_{ordered}$, obtained from the least-square fit to the data in Figs. 2(b) and (d), also agree with each other. Note the close agreements between fitted and predicted values of the allometric exponents γ in all cases

| | H vs M | H vs M/H^3 | H vs M/H^2 | H vs $M/H^{\gamma_{th}}$ | H vs $M/H^{\gamma_{ex}}$ |
|---------------------------|------------|----------------|----------------|----------------------------|----------------------------|
| Pearson's r (Hong Kong) | 0.50 | -0.43 | -0.12 | 0.01 | 0.01 |
| Pearson's r (Sweden) | 0.64 | -0.21 | 0.14 | 0.03 | 0.02 |

Table 2. Pearson's correlation coefficient r between the height H and various quantities (M , M/H^3 , M/H^2 , $M/H^{\gamma_{th}}$, and $M/H^{\gamma_{ex}}$) are computed for Hong Kong children [13] (first row) and the Swedish data [14] (second row). The height-mass correlation (r for H vs M) is of course positively significant in both cases. For the Hong Kong data in the first row the correlations between H and M/H^3 , and between H and M/H^2 are negative, implying that the exponents 2 and 3 are overestimations. In contrast, $M/H^{\gamma_{ex}}$ and $M/H^{\gamma_{th}}$ exhibit neutral correlation with H , which shows that our estimation $\gamma_{th} \approx 1.63$ describes data much better than the conventional BMI-value $\gamma = 2$. Likewise in the second row for the Swedish data the correlations between H and M/H^3 , and between H and M/H^2 are, respectively, negative and positive, implying an exponent between 2 and 3. This is again in agreement with our analysis and theory.

gives a direct demonstration of the connection between spread and power-law exponent.

Table 2 gives various Pearson's r -coefficients computed for various pairs of quantities. For the Hong Kong children data (first row of the table) the height-mass correlation (H vs M) is significantly positive, implying that in general the taller child has the heavier mass. It is to be noted that the correlations for M/H^3 and M/H^2 deviate much from zero, while $M/H^{\gamma_{ex}}$ and $M/H^{\gamma_{th}}$ exhibit neutral correlation with the height, suggesting that our estimation $\gamma_{th} \approx 1.63$ describes data much better than the conventionally used BMI value $\gamma = 2$.

In order to rule out that there is anything accidental or fortuitous about the results presented, we have investigated a second data-set in the same way. This second data set gives height and mass for Swedish children between 13.5 to 19 years old (more precisely between 5000 to 7000 days old containing in total 11327 data points) [14]. The results are presented in Fig. 2(c) and (d) with parameters given in the second rows of Tables 1 and 2. From Table 1 one can see that the average height and mass for these two data-sets are roughly the same. However, since the Swedish children data spans over a longer

age period than the Hong Kong data, the standard deviations for height, $\tilde{\sigma}_H$, and mass, $\tilde{\sigma}_M$, are larger by about a factor 2. This is of course because children during a longer period grows more. Yet the ratio between the standard deviations, $\tilde{\sigma}_M/\tilde{\sigma}_H$, is closely equal for the two data-sets (3.26 and 3.25 respectively.) This ratio is in fact the $\gamma_{\sigma=0}$ and, as seen from Table 1 and Figs. 2(b) and (d), $\gamma_{\sigma=0}$ gives very precise estimates of the allometric exponents γ_{ordered} . The fact that the exponents γ_{ordered} are very nearly the same for the two data-sets, suggests that, in this particular aspect, children from Hong Kong and Sweden are very similar. Also for the Swedish data set there is a good agreement between the experimental allometric exponent γ_{ex} and the prediction γ_{th} from Eq. (10) (compare Figs. 2(c) and Table 1). However, there is a significant difference between the allometric exponents γ_{ex} for the two data-sets: $\gamma_{ex} = 1.63$ and 2.35 for, respectively, Hong Kong and Swedish children. The close agreement between γ_{ex} and the prediction γ_{th} for both data-sets suggests that the difference in value of the allometric exponent γ_{ex} can be attributed to a relatively larger spread in weight for the children of average height for Hong Kong children compared to Swedish children. From this point of view it is rather a sampling difference than some difference in trait of a Hong Kong and Swedish individual. A possible explanation could be that, since Hong Kong compared to Sweden for a long time has been a human hotspot with influx of people of great variety in both genetical and cultural backgrounds, this has resulted in a relatively larger spread of weight for a given height in a particular age interval.

4. Discussion

The implication of theses results become clearer when comparing Fig. 2(a) and Fig. 2(b). Both represent data with the same two Gaussian distributions for mass and height given in Fig. 1. The difference is that the data in Fig. 2(a) also has a spread of mass for individuals with a given height, as shown in Fig. 1. For the artificial data in Fig. 2(b) there is no such spread. The data in Fig. 2(b) represents a true allometric relation between mass and height of the form $M \propto H^\gamma$: as soon as you pick a person with a certain height you also to very good approximation know his mass. Since γ for this artificial dataset is 3.29, it either means that these artificial people either all have the same shape but get a bit denser with increasing height, or that they just become somewhat disproportionally fatter with increasing height. The point is that you in this case can relate the allometric exponent to some property of the individual. However, for the real data in Fig. 2(a) this becomes more problematic. This is because for a given height the individuals have a random mass distributed about the mean, as shown in Fig. 1(c). This random spread can have a multitude of different causes, like availability of food, climate, diseases, genetics etc. Different individuals are affected by these multitude of causes in different ways.

An alternative way of describing this is as follows: Suppose you have a dataset like the one discussed in the present paper and suppose that each *individual* can be characterized by the allometric relation $M \propto \langle H \rangle_M^{\gamma_0}$, where $\langle H \rangle_M$ is the average height

for individuals of mass M . This means that if you pick an individual with mass M then you know that his most likely height is given by allometric relation with γ_0 . If in addition there was no randomness in the height, then you for certain know his height and the result is given by Fig. 2(b). But if there is an randomness in the height caused by many causes, so that the probability of the height for an individual is described by a Gaussian probability distribution, then the allometric exponent becomes smaller and you can end up with something like Fig. 2(a) instead. The difference is that Fig. 2(a) describes the *collective* dataset, whereas Fig. 2(b) corresponds to an allometric relation on an *individual* basis.

In Fig. 2 (a) and (b) it is precisely the random spread which causes the decrease of the allometric exponent from 3.29 to 1.63. So you cannot any longer associate the allometric exponent γ with some unique growth property of the individuals. It is rather like that the spread in Fig. 1(c) just tells you how much the masses and heights for individuals are random and uncorrelated. This is also reflected by the prediction given by Eq. (10) which decreases from its maximum value $\gamma = \frac{\tilde{\sigma}_M}{\tilde{\sigma}_H}$ to zero with increasing random spread $\tilde{\sigma}$, as illustrated in Fig. 3.

Some further insight to this is given by the comparison between the Hong Kong children and the Swedish children: The allometric exponent for the Hong Kong children is significantly smaller than for the Swedish children. According to the present analysis, this difference can be traced to the difference in the relatively larger spread in mass for a given height in case of the Hong Kong data-set. One possible explanation for this difference in spread could be a sampling difference: Compared to Sweden, Hong Kong has been a historical hotspot leading to a greater variety of people both from a genetical and a cultural point of view. Such greater variety is also likely to cause a larger variety in weight for a given height.

The present analysis is quite general and its implication is likely to have a wider range of applicability within allometric relations than just to the illustrative example of mass-height relation for humans, discussed here: It brings a caution against attributing too much specific cause to the precise value of an allometric exponent. The crucial point is that the allometric exponent for an individual is, because of randomness, not the same as the allometric exponent of the collective dataset.

Acknowledgments

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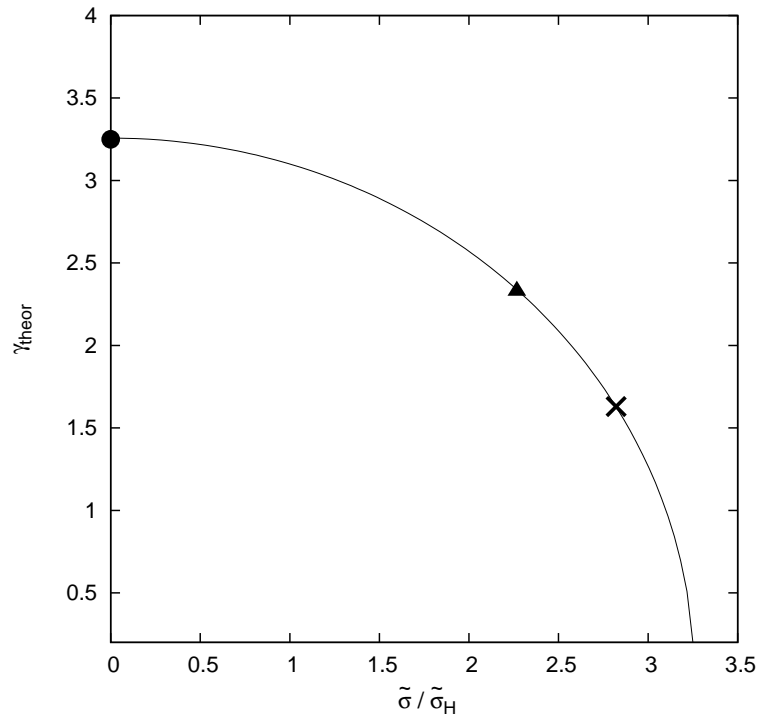


Figure 3. Plot of the γ_{th} obtained from Eq. (10). The standard deviations $\tilde{\sigma}_M$ and $\tilde{\sigma}_H$ from the children data are used to plot γ_{th} as a function of the standard deviation ratio $\sigma/\tilde{\sigma}_H$. Note that, since $\gamma_{\sigma=0} = \tilde{\sigma}_M/\tilde{\sigma}_H$ is almost identical for the Hong Kong and Swedish data, both predictions are obtained from the same curve. The black dot represent the predicted maximum value $\gamma_{\sigma=0}$ for $\sigma = 0$. The cross and triangle represents the prediction of γ_{th} for, respectively, the Hong Kong and Swedish children data.

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